LSI - Filter

2D Faltung:

$$I * H(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(x-\lambda_1, y-\lambda_2)\, H(\lambda_1,\lambda_2)\, d\lambda_1\, d\lambda_2$$

diskrete Faltung:

$$I * H(m,n) = \sum_{i=-\infty}^{\infty}\sum_{j=-\infty}^{\infty} I(m-i, n-j)\, H(i,j)$$

↳ Kommutativität $\mathcal{H}_1 \mathcal{H}_2 = \mathcal{H}_2 \mathcal{H}_1$

↳ Assoziativität $(\mathcal{H}_1 \mathcal{H}_2)\mathcal{H}_3 = \mathcal{H}_1(\mathcal{H}_2 \mathcal{H}_3)$

↳ Distributivität bei Addition $(\mathcal{H}_1 + \mathcal{H}_2)\mathcal{H}_3 = \mathcal{H}_1\mathcal{H}_3 + \mathcal{H}_2\mathcal{H}_3$

LSI = Linear Shifting Invariante

Verschiebungsinvarianz

$$\mathcal{H}\, {}^{mn}S = {}^{mn}S\, \mathcal{H}$$

$$\boxed{{}^{mn}S\, g_{m'n'} = g_{m'-m,\, n'-n}}$$

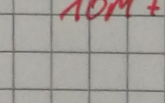
↑ Verschiebungsoperator ${}^{mn}S$

Gaußoperator zum Weichzeichnen

Faltungskern:

$$\begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix} = [1\ 4\ 6\ 4\ 1] * \begin{bmatrix} 1 \\ 4 \\ 6 \\ 4 \\ 1 \end{bmatrix}$$

25 M + 24 ADD PP



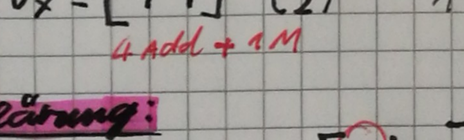
Besser zerlegen

5 M + 4 Add PP

⇓

10 M + 8 Add PP

$$\boxed{I' = H_G * I}$$



[1 4 6 4 1] auch als:

$$B_x^4 = [1\ 1]^4 \cdot \left(\tfrac{1}{2}\right)^4$$

4 Add + 1 M

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    1 1
   1 2 1
  1 3 3 1
 1 4 6 4 1
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Erklärung:

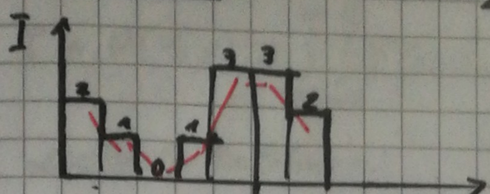
$I' = H * I$ $\qquad H = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ $\qquad I = \begin{bmatrix} \cdots & & \cdots \\ & \boxed{\;\cdot\;} & \\ \cdots & & \cdots \end{bmatrix}$

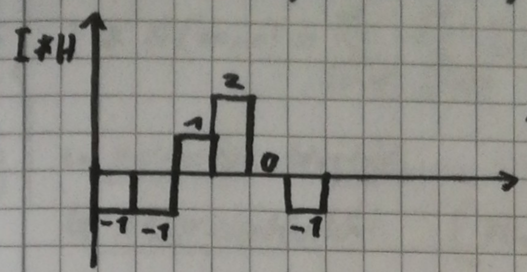
$$I'(m,n) = H * I(m,n)$$

$$I'_{mn} = \sum_{j=-1}^{+1}\sum_{i=-1}^{+1} H(i,j) \cdot I(m-i, n-j)$$

$$= (a \cdot I_{m+1,n+1} + b \cdot I_{m,n+1} + c \cdot I_{m-1,n+1}$$
$$+ d \cdot I_{m+1,n} + e \cdot I_{m,n} + f \cdot I_{m-1,n}$$
$$+ g \cdot I_{m+1,n-1} + h \cdot I_{m,n-1} + i \cdot I_{m-1,n-1})$$

Beispiel: 1D-Faltung





H → Faltungskern $\boxed{1\ |\ -1\ |\ 0}$

$1 + 2 + 0$

$1 - 2 + 0 = -1$

Ableitungsoperator

$$\frac{df}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\Downarrow$$

$$\frac{dI}{dm} = \frac{I(m+1) - I(m)}{1}$$

$$= I(m+1) - I(m)$$

$$= 1 \cdot I_{m+1} + (-1) \cdot I_m$$

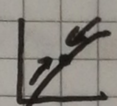
$d_x^+ = \boxed{1\ |\ -1\ |\ 0}$

der von unten kommt:

$d_x^- = \boxed{0\ |\ 1\ |\ -1}$

Bessere Ableitung:

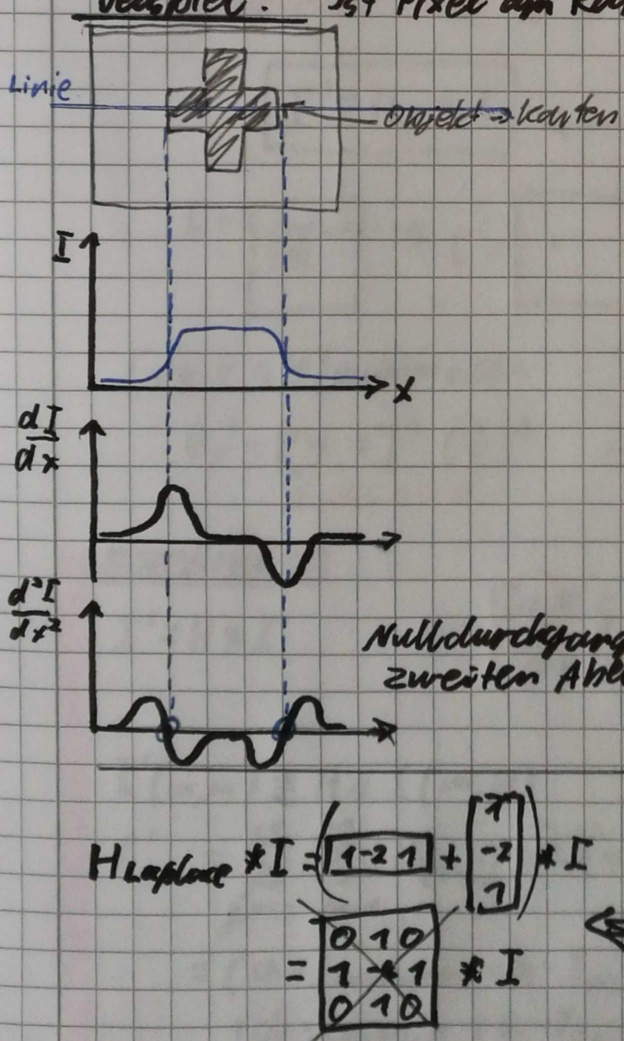
$$\left.\frac{df}{dx}\right|_{+h} + \left.\frac{df}{dx}\right|_{-h}$$



$$\frac{d_x^+ + d_x^-}{\cancel{2}} = \begin{array}{|c|c|c|}\hline 1 & -1 & 0 \\ \hline 0 & 1 & -1 \\ \hline \end{array} = \boxed{1\ |\ 0\ |\ -1} \left(\cdot \frac{1}{2}\right)$$

egal

Beispiel: Ist Pixel am Rand eines Objektes



Nulldurchgang der zweiten Ableitung

2. Ableitung bilden

1. Abl: D_x^+ $\boxed{1\ |\ -1}$

$$D_x^+ * (D_x^+ * I(x))$$

$$= (D_x^+ * D_x^+) * I(x)$$ Assoziativ

$$= \left(\boxed{1\ |\ -1} * \boxed{1\ |\ -1}\right) * I(x)$$

$\boxed{-1\ |\ 1} \rightarrow$

$$= \boxed{1\ |\ -2\ |\ 1} * I(x)$$

2-Dimensional

$$\vec{\nabla} f(x,y) = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix}$$

$$\vec{\nabla} = \begin{pmatrix} \partial/\partial x \\ \partial/\partial y \end{pmatrix}$$

Laplace

$$\Delta f = \vec{\nabla}\vec{\nabla} f(x,y) = \begin{pmatrix} \partial/\partial x \\ \partial/\partial y \end{pmatrix} \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix}$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right)$$

$$= \frac{\partial^2}{\partial x^2} f + \frac{\partial^2}{\partial y^2} f$$

$$H_{Laplace} * I = \left(\boxed{1\ |\ -2\ |\ 1} + \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}\right) * I \Leftarrow$$

$$= \begin{array}{|c|c|c|}\hline 0 & 1 & 0 \\ \hline 1 & -4 & 1 \\ \hline 0 & 1 & 0 \\ \hline \end{array} * I$$

nicht gut in Diagonale

Besserer Laplace Operator

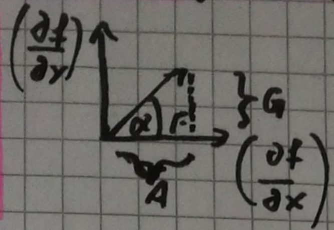
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix} = H_{\text{Laplace optimiert}}$$

Länge des Vektors:

$$|\nabla f(x,y)| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

„Länge"

Richtung des Vektors:



$$\tan(\alpha) = \frac{\left(\frac{\partial f}{\partial y}\right)}{\left(\frac{\partial f}{\partial x}\right)} \Rightarrow \arctan$$

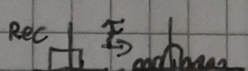
⇒ **Achtung natürliche Bilder haben Rauschen, somit ganz viele 0-Durchgänge in 2. Abl.**

Gegenmaßnahme:

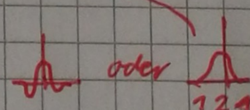
Mitteln

$$\frac{1}{3}\begin{array}{c} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \end{array}\begin{array}{c} 1 \\ 2 \\ 1 \end{array} * \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} = \text{Prewitt}$$

Rechteck ist doof



Besser



Sobel

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

Operator in x-Richtung